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On the Position of λ Ursæ Minoris.

By Professor T. H. Safford.

The star in question is extremely well known; but its proper motion and nearness to the Pole make it very troublesome to predict its place; so that it furnishes a good example of the treatment of proper motions in that region of the heavens.

I begin with the assumption of the following formula, as representing its position for time t referred to the Equator and Equinox of 1855:—

$$\text{A.R.} = 20^{\text{h}}\ 8^{\text{m}}\ 30^{\text{s}}{\cdot}200 - 0^{\text{s}}{\cdot}073\,(t - 1855),$$

$$\text{Decl.} = 88^{\circ}\ 52'\ 32''{\cdot}30\ + 0''{\cdot}01\ (t - 1855).$$

These were derived from a first approximate solution, after reducing to 1855 the material most conveniently at hand.

The first step is to calculate for various epochs the positions which correspond to this formula, after application of precession to the epochs in question.

With the position for 1855,

$$\alpha = 20^{\text{h}}\ 8^{\text{m}}\ 30^{\text{s}}{\cdot}200,$$

$$\delta = 88^{\circ}\ 52'\ 32''{\cdot}30,$$

and the following values of the precession-constants $z+\lambda$, $z'-\lambda'$, θ, derived from the data of Bessel's *Tabulæ Regiomontanæ*:—

	$z+\lambda$ ′ ″	$z'-\lambda'$ ′ ″	θ ′ ″
1755	−38 25·02	−38 19·54	−33 25·869
1790	25 0·01	24 53·31	21 43·786
1800	21 10·03	21 2·84	18 23·123
1810	17 20·07	17 12·35	15 2·536
1815	15 25·09	15 17·09	13 22·245
1835	7 45·21	−7 35·94	−6 41·104
1855	−5·37	+5·37	0·000
1875	+7 34·42	7 46·85	+6 41·065

By the help of these formulæ,

$$A = \alpha + z + \lambda,$$

$$m \cos M = \cos\delta \cos A,$$

$$m \sin M = \sin\delta,$$

$$\cos\delta' \sin A' = \cos\delta \sin A,$$

$$\cos\delta' \cos A' = m \cos(M+\theta),$$

$$\sin\delta' \qquad = m \sin(M+\theta),$$

$$\alpha' = A' + z' - \lambda',$$

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were obtained the following values of α', δ', the right ascensions and declinations for the various equinoxes and equators, corresponding to the fixed values for 1855 :—

	α'			δ'		
	°	′	″	°	′	″
1755	319	24	5·34	88	30	25·24
1790	314	28	52·1	88	38	58·86
1800	312	52	28·4	88	41	17·41
1810	311	10	7·2	88	43	31·70
1815	310	16	36·49	88	44	37·13
1835	306	26	4·54	88	48	46·21
1855	302	7	33·00	88	52	32·30
1875	297	19	34·67	88	55	51·35

	A.R. in Time.		
	h	m	s
1755	21	17	36·356
1790	20	57	55·47
1800	20	51	29·89
1810	20	44	40·48
1815	20	41	6·433
1835	20	25	44·303
1855	20	8	30·200
1875	19	49	18·311

The values of the precession, secular variation, and Argelander (Tiele's) third term were then computed for the last four dates, as follows :—

	In A.R.		
	Precession.	Secular Variation.	Third Term.
	s	s	s
1815	−43·4488	−25·772	−209·2
1835	−48·8398	−28·055	−165·1
1855	−54·6192	−29·562	− 77·0
1875	−60·5727	−29·678	+ 67·5

In computing these quantities, the tables by Menten and Tiele in vol. vii. of the *Bonn Observations*, and those there referred to in the *Washington Observations* for 1847, were employed; the quantities being modified where necessary to correspond to Bessel's m and n from the *Tabulæ Regiomontanæ*.

In calculating the secular variations, I found it necessary to compute the coefficient of $\tan^2 \delta$ to more decimals than the table gives. This coefficient is

For 1755 [8·11432] sin 2α
1875 [8·11381] sin 2α.

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Menten's table gives to four places the logarithm of [8·1139] sin $2a$.

The similar coefficients in declination are

	Precession.	Secular Variation.	Third Term.
	″	″	″
1815	+12·9671	−4·838	−69·2
1835	11·9115	−5·734	−79·8
1855	10·6604	−6·751	−88·7
1875	+ 9·2051	−7·752	−93·8

The third term, as calculated by the help of Tiele's table, is not to be depended upon in its last place at so high northern declinations; but as it is the coefficient of $\left(\frac{t}{100}\right)^3$ in the development according to powers of the time, this does not matter much for an interval of ten years before or after the epoch.

Multiplied by ·06 this "third term" gives us the secular variation of the secular variation, as can be easily shown by Taylor's theorem, and roughly controlled by the values given.

Of more consequence is sometimes the fourth term in the development. If we wish to proceed, say from 1835 to 1845, we can interpolate the third term roughly for one-fourth the interval; or in this case from 1835 to 1837·5; which gives us -156^{s} for its value at the last date. Using this value we shall take into account the fourth term so far as is needful. Then the A.R. for 1845 will be

	h	m	s
A.R. 1835	20	25	44·303
10 × Precession		−8	8·398
$\frac{1}{2}$ × Secular Variation			−14·027
$\left(\frac{10}{100}\right)^3$ × Third Term			−0·156
A.R. 1845	20	17	21·72

The next step is to compute the effect of proper motion, assumed as $-0^{s}{\cdot}073$ and $+0''{\cdot}01$ annually, referred to the planes of 1855.

To refer them to any other plane the formulæ will be

$$\sin \xi = \frac{\sin \theta \sin A'}{\cos \delta},$$

where A' is $a'-z'+\lambda'$ before used, and δ is the declination for 1855.

Then if

$$\begin{aligned} \Delta\alpha &= -0^{s}{\cdot}073\,(t-1855) = -1''{\cdot}095\,(t-1855), \\ \Delta\delta &= +0''{\cdot}01\,(t-1855), \end{aligned}$$

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we shall have

$$\Delta\alpha' \cos\delta' = \Delta\alpha \cos\delta \cos\xi + \Delta\delta \sin\xi,$$
$$\Delta\delta' = -\Delta\alpha \cos\delta \sin\xi + \Delta\delta \cos\xi.$$

These formulæ give us the following results:—

	$\Delta\alpha'$ ″	$\Delta\delta'$ ″	ξ °	′
1790	+51·41	−0·952	+13	11·44
1800	+45·82	−0·774	+11	27·32
1810	+39·47	−0·605	+ 9	37·27
1815	+36·00	−0·525	+ 8	39·90
1825	+28·41	−0·373	+ 6	40·29
1835	+19·91	−0·233	+ 4	33·94
1845	+10·45	−0·109	+ 2	20·59
1855	0·00	0·000		
1865	−11·46	+0·091	− 2	27·97
1875	−23·89	+0·161	− 5	3·39
1885	−37·25	+0·210	− 7	46·24

For 1755 the values of $\Delta\alpha'$ and $\Delta\delta'$, $+65''{\cdot}96$ and $-1''{\cdot}63$ respectively, were obtained by a direct computation, using

$$\alpha = 302^\circ\ 9'\ 22''{\cdot}50$$

and

$$\delta = 88^\circ\ 52'\ 31''{\cdot}30.$$

The next step is the comparison of these computed places with observation, carried out as follows:—

	Computed A.R.	Computed Decl.	
	° ′ ″	° ′ ″	
1790	314 28 52·1	+88 38 58·86	With precession alone.
	+51·41	−0·95	Effect of proper motion.
	314 29 43·51	+88 38 57·91	Sum: Computed place.
	h m s		
	20 57 58·90		
Fedorenko's Catalogue	20 58 2·38	88 38 57·76	Observed place.
Computed—Observed	−3·48	+0·15	

In this case the proper motion from the date of observation to the epoch is neglected.

In the following table are given the comparisons for which I have materials at hand. The principal observations wanting are Piazzi, Greenwich, 1868–1872, and various modern series not yet combined into Catalogues; a few positions here given are derived at second-hand from former investigations by myself or others; and several of the places in the table need revision and correction for the adopted nutation:—

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	$\Delta\alpha$	$\Delta\alpha \cos\delta$	$\Delta\delta$	No. Obs.
	s	″	″	
Bradley 1755	−0·79	−0·31	+0·21	3
Fedorenko 1790*	−3·48	−1·23	+0·15	2
Groombridge 1807*	+1·75	+0·58	+0·8	Error in Radcliffe?
Struve 1815*	+0·4	+0·13	...	4
Schwerd 1828*	0·0	0·00	−0·10	11
Catalogus Aboensis 1830	+0·04	+0·01	+0·25	17
Cambridge 1832	+3·67	+1·15	...	14
Pond 1833	−0·77	−0·24	+0·25	20, 24
Cambridge 1835	...	...	+0·20	17
Taylor 1835	+0·97	+0·30	+1·13	2, 3
Henderson 1838	−0·35	−0·11	−0·08	26, 39
Challis 1838	...	...	+0·30	32
Airy 1840	−1·54	−0·47	+0·05	25, 77
Armagh 1843	+0·56	+0·17	...	36
Radcliffe 1843	...	...	+0·48	158
Airy 1845	−1·55	−0·47	+0·21	15, 56
Pulkova 1845	+0·17	+0·05	+0·16	21, 5
Radcliffe 1847	−0·26	−0·08	...	111
Airy 1851	−1·27	−0·38	−0·08	49, 106
Armagh 1851	...	...	−0·26	5
Carrington 1855	+0·3	+0·09	−0·3	27
Yarnall 1856	...	...	+0·18	117
Airy 1857	−0·72	−0·21	+0·19	80, 123
Paris 1857*	+0·23	+0·07	−0·24	50, 59
Airy 1864	+0·40	+0·12	+0·24	30, 84
Pulkova 1868	+0·24	+0·07		
Leiden 1868	...	...	+0·14	40
Washington 1870	−0·14	−0·04	+0·19	104, 177
Rogers 1872	−0·84	−0·24	+0·18	20, 12
Airy 1874	+0·95	+0·27	−0·16	36, 91

The authorities marked with an asterisk are taken at second-hand from others' investigations or my own. There seems to be an error of $1''$ to $2''$ in the reduction of Groombridge's Polar Distance to 1845, as found in the Radcliffe Catalogue.

To reduce the values of $\Delta\alpha$ to the parallel the following little table of log ($15 \cos \delta$) was found serviceable:—

Year.	(log 15 cos δ.)	Year.	(log 15 cos δ.)
1755	9·592	1835	9·492
1790	9·548	1855	9·469
18	9·516	1875	9·447

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My first intention was to assign weights and proceed to a solution of equations of condition, thus correcting the assumed position; but the resulting corrections were found to be very small, and I have judged it best to wait until I can add all the observations to be found in Catalogues which I do not possess. The experiments which I made led mainly to the conclusion that the formulæ heretofore given cannot be much improved at present, at least with any certainty.

In carrying the calculations forward for the next few years, the following values for 1885, depending on the same elements, will be found useful:—

	A.R.			Decl.		
	h	m	s	°	′	″
Value without P.M. from 1855	19	38	57·84	+88	57	19·39
Annual Precession			−63·5110			+8·3917
Secular Variation			−28·998			−8·415
Third Term			+162·2			−93·6

The effect of P.M. from 1855 up to the year of observation should be interpolated from the previous table and added to the values derived from these numbers.

In case the arrangement of the calculation given in this paper shall save anyone else the annoyances which I have undergone in the solution of similar problems, my object will have been gained; the points of originality in the process are of course few and small.

The star's place itself would seem not to require so many observations as have been made for a few years, but rather such as shall individually possess the greatest possible accuracy.

Williams College,
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May 15, 1878.

A Reply to Mr. Proctor's Note in the March Number of the Monthly Notices.

By E. J. Stone, M.A., F.R.S., Her Majesty's Astronomer, Cape of Good Hope.

1. Mr. Proctor's remarks in the March No. on my paper in the January No. appear to call for some reply on my part.

In Mr. Proctor's paper in the Supplementary No. of the *Notices*, 1877, he says that he has proved "that by no optical contrivances can apparent brightness be increased, though quantity of light can, of course, be greatly increased." I stated in my note in the January No. that this statement requires qualification; for, if it were true, why could not the satellites of *Mars* be as easily seen with a 4-inch telescope as with the 26-inch of the Washington Observatory? Mr. Proctor's answer to this is that my question contains its own answer, viz. "that quantity of light is, of course, greatly increased when the larger telescope is used." But the satellites would not be visible with the large telescope rather than with the small one unless their apparent